Seat No. : _______________

ZS-111

May-2014

M.Sc., Sem.-II

410 : Mathematics (Partial Differential Equations)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Attempt any **one** : **7**

(i) Find the general integral of $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = 2(x^2 + y^2)z$.

(ii) Verify that the Pfaffian differential equation

$$(1 + yz)dx + z(z - x)dy - (1 + xy)dz = 0$$

is integrable and find its integral.

(b) Attempt any **two** : **4**

(i) Eliminate the arbitrary function F from the equation below and find the corresponding p.d.e.

$$F\left(\frac{xy}{z}, \frac{x - y}{z}\right) = 0$$

(ii) Show that $z = ax + (y/a) + b$ is a complete integral of $pq = 1$. Find the particular solution corresponding the sub-family $b = a$.

(iii) Prove : If the Pfaffaian differential equation

$$P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$$

is integrable, then $\vec{X} \cdot \text{curl}\, \vec{X} = 0$, where $\vec{X} = (P, Q, R)$.

(c) Answer very briefly : **3**

(i) Form a partial differential equation by eliminating the parameters a and b from $z = ax + by$.

(ii) Is the equation $(6x + yz)dx + (xz - 2y)dy + (xy + 2z)dz = 0$ integrable ? Why ?

(iii) Obtain the envelope of the family of spheres $x^2 + y^2 + (z - a)^2 = 1$.

2. (a) Attempt any **one** : **7**

(i) Find a complete integral of the equation $p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$.

(ii) Find the complete integral of the equation $p^2x + qy - z = 0$ and derive the equation of the integral surface containing the line $y = 1$, $x + z = 0$.

(b) Attempt any **two** : **4**

(i) By Jacobi's method, solve the equation $xu_x + yu_y = u_z^2$.

(ii) Solve the Cauchy problem for $2z_x + yz_y = z$ for the initial data curve

$C : x_0 = s,\ y_0 = s^2,\ z_0 = s,\ 1 \le s \le 2.$

(iii) Find a complete integral of $zpq = p^2q(x + q) + pq^2(y + p)$.

(c) Answer very briefly : **3**

(i) What is the complete integral of the equation $u_x + u_y + u_z - u_xu_yu_z = 0$?

(ii) Find a complete integral of $p + q = pq$.

(iii) What is the complete integral of the equation $p^2 + q^2 = x + y$?

3. (a) Attempt any **one** : **7**

(i) Find by the method of characteristics, the integral surface of $pq = xy$ which passes through the line $z = x$, $y = 0$.

(ii) Reduce the equation

$$u_{xx} - 4x^2u_{yy} = \frac{1}{x}u_x$$

to a canonical form.

(b) Attempt any **two** : **4**

(i) Classify the equation $e^{2x}u_{xx} + 2e^{x+y}u_{xy} + e^{2y}u_{yy} = 0$.

(ii) Find the possible initial strips for the solution of the equation $z = p^2 - q^2$ passing through the curve $C : x_0 = s,\ y_0 = 0,\ z_0 = -\frac{1}{4}s^2$.

(iii) Transform the one-dimensional wave equation

$$y_{xx} = \frac{1}{c^2}y_{tt}, \quad -\infty < x < \infty,\ t > 0$$

to a canonical form using the characteristic variables $\xi = x - ct$, $\eta = x + ct$.

(c) Answer very briefly : **3**

(i) Write down the d'Alembert's solution of one-dimensional wave equation,

(ii) State the problem of vibrations of a semi infinite string.

(iii) Give an example of a second order p.d.e. which is of elliptic type.

4. (a) Attempt any **one** : **7**

(i) Prove that the solution of the following problem, if it exists, is unique.

$$u_{tt} - c^2 u_{xx} = F(x, t), \ 0 < x < l, \ t > 0,$$

$$u(x, 0) = f(x), \ 0 \le x \le l,$$

$$u_t(x, 0) = g(x), \ 0 \le x \le l,$$

$$u(0, t) = u(l, t) = 0, \ t \ge 0.$$

(ii) State and prove Maximum principle.

(b) Attempt any **two** : **4**

(i) Prove that the solution of Dirichlet problem, if it exists, is unique.

(ii) Discuss the stability of the solution of the following problem :

$$u_{xx} + u_{yy} = 0$$

$$u(x, 0) = 0$$

$$u_y(x, 0) = n^{-1} \sin nx$$

The solution of this problem is $u(x, y) = n^{-2} \sinh ny \sin nx$.

(iii) State interior and exterior Dirichlet Problems for a circle.

(c) Answer very briefly : **3**

(i) State Dirichlet problem for the upper half plane.

(ii) What is well-posed problem ?

(iii) State Dirichlet problem for a rectangle.

5. (a) Attempt any **one** : **7**

(i) Solve the following problem :

$$u_t = ku_{xx}, -\infty < x < \infty, t > 0,$$

$$u(x, 0) = f(x), -\infty < x < \infty.$$

(ii) Show that the family of surfaces

$$x^2 + y^2 + z^2 = cx^{2/3}$$

can form an equipotential family of surfaces, and find the general form of the potential function.

(b) Attempt any **two** : **4**

(i) Prove that the solution of the Neumann problem is unique up to the addition of a constant.

(ii) State (only) Harnack's theorem.

(iii) State Neumann problem for the upper half plane.

(c) Answer very briefly : **3**

(i) State a condition under which one-parameter family of surfaces

$f(x, y, z) = c$ forms equipotential surfaces.

(ii) Give an example of a harmonic function.

(iii) State Heat conduction problem for a finite rod.